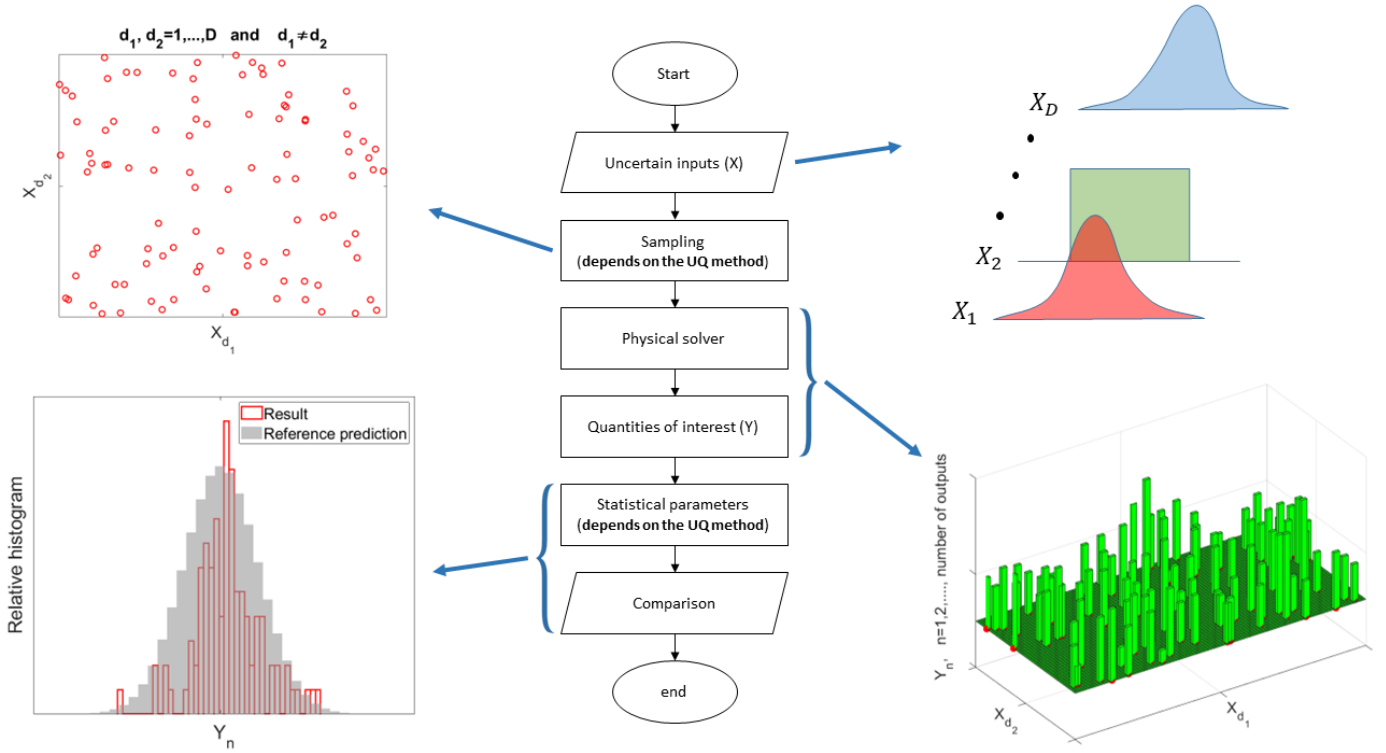
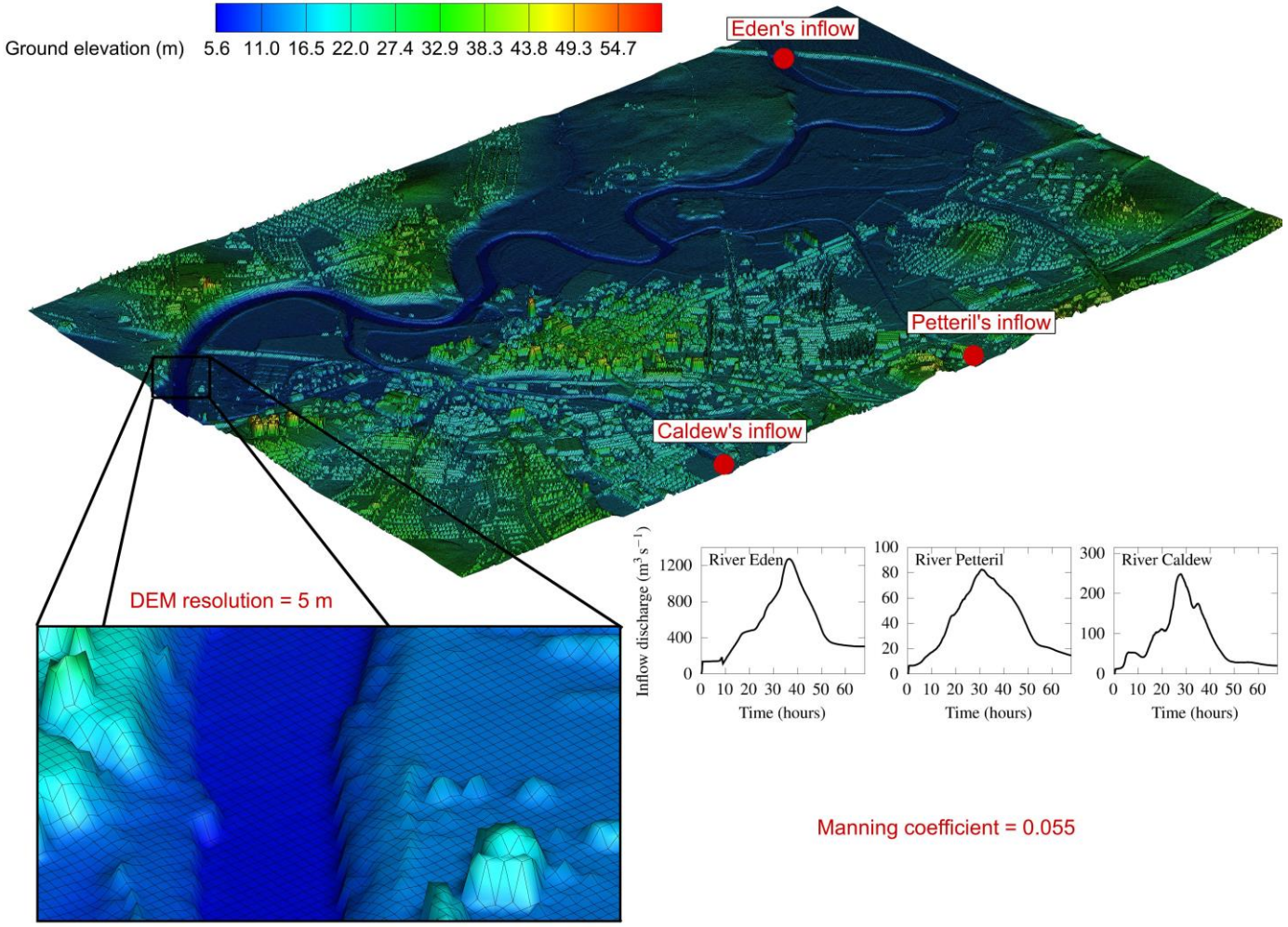
Efficient uncertainty quantification methods for flood modelling

Mahya Hajihassanpour

Research associate, University of Sheffield



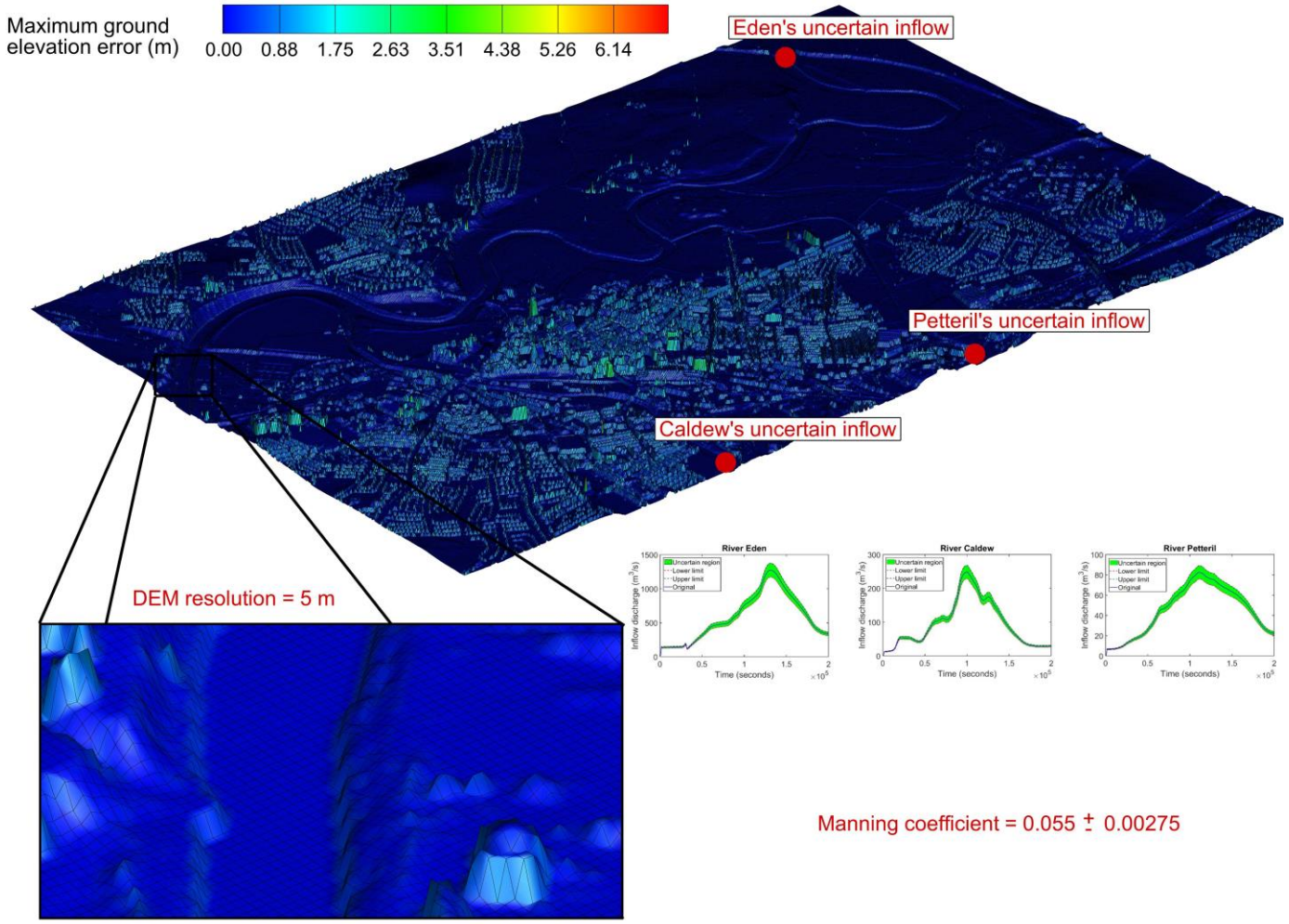
Uncertainty quantification



- Inputs:
  - Inflow discharges
  - Manning coefficient
  - Ground elevation (DEM)
- Outputs:
  - Water depth (h)
  - Velocity field (v)
- Quantities of interest:
  - Flood extent
  - Hazard rates ($HR=(V+0.5)\times h$)
    - HR_{ave}
    - HR_{max}

[UK Environment agency]

Uncertainty quantification



- Inflow discharges ($\pm 8\%$)
- Manning coefficient ($\pm 5\%$)
- Ground elevation ($\left|cMl_d^2\right|$)

User-defined constant ~ 0.1

Local second order spatial derivative magnitude

DEM's resolution

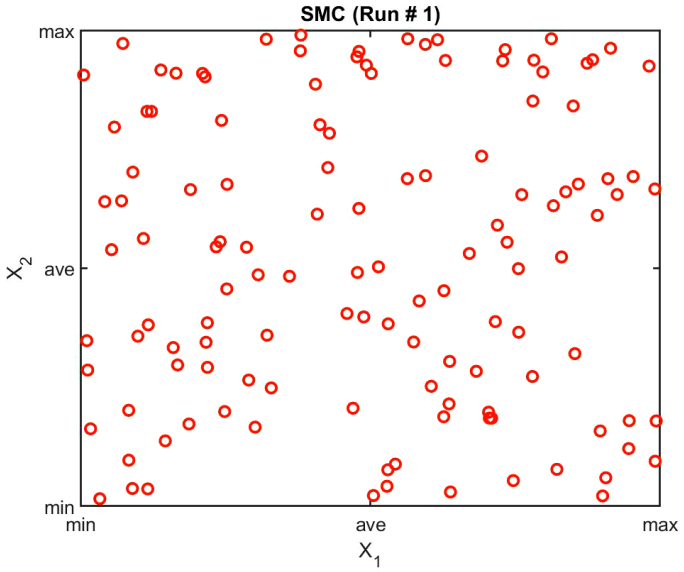
quantities of interests

Statistical property

Histogram

Purpose

- Standard Monte Carlo (SMC)



$$\text{Error} \sim N_s^{-1/2}$$

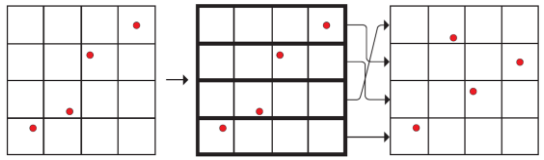
- **Replications**
- **Large sample size**

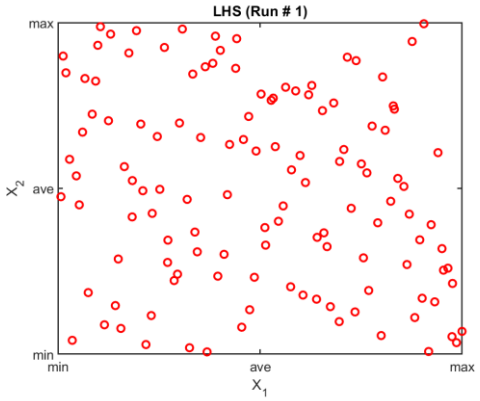
- **Purpose: Finding the best alternative to SMC**
  - Any type of the histogram
  - Smaller sample size than SMC (within 10% error)

Alternative UQ methods

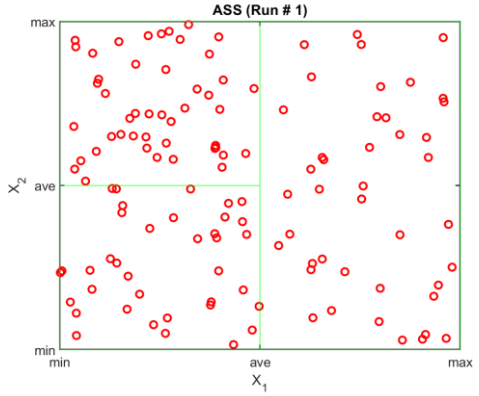
Random sampling methods

✓ Latin Hypercube Sampling (LHS)



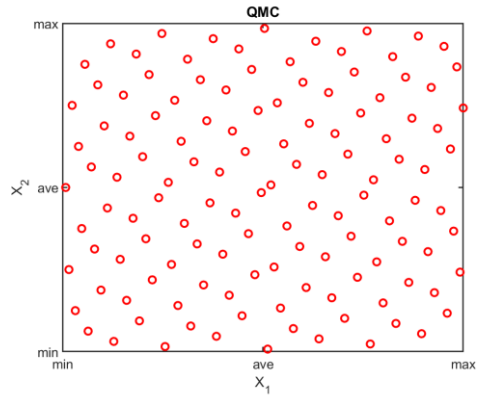


✓ Adaptive Stratified sampling (ASS)

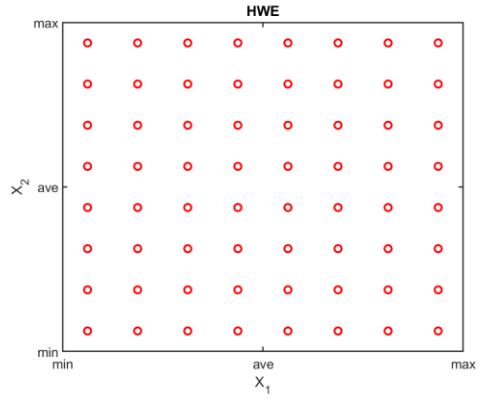


Deterministic realisation methods

✓Quasi Monte Carlo (QMC)



✓Haar Wavelet expansion (HWE)



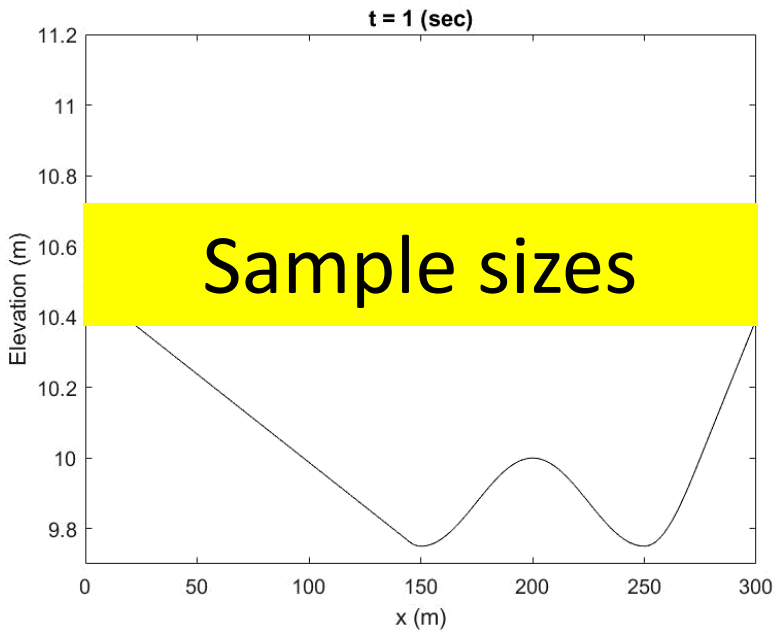
Number of dimensions

$$N_s = 2^{DL}$$

Refinement level

Benchmarking test cases

1) Rapidly propagating flood

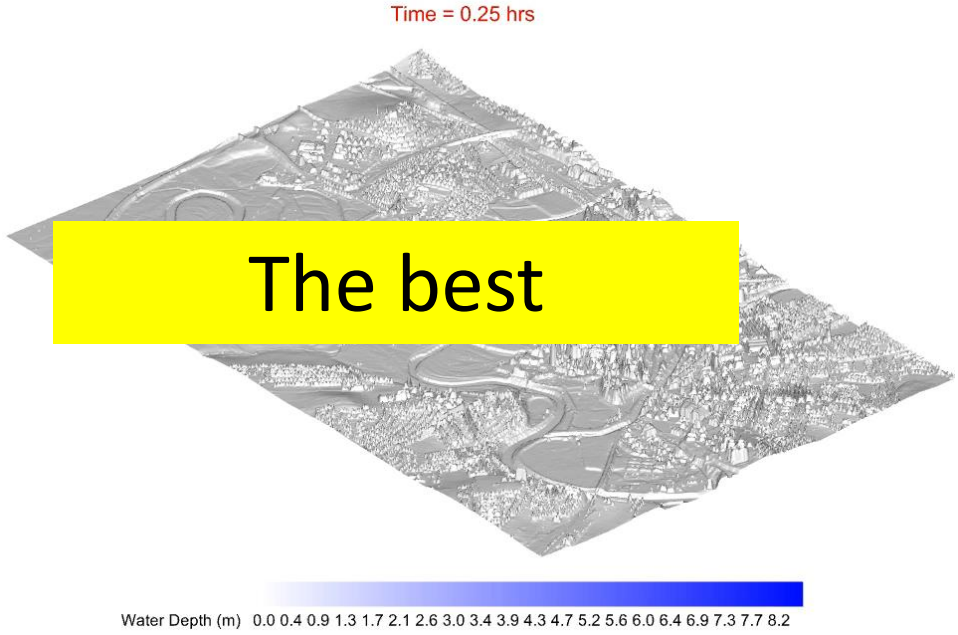


Sample sizes

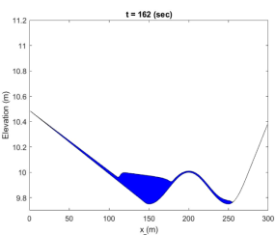
2) Flooding in a valley

Two best methods

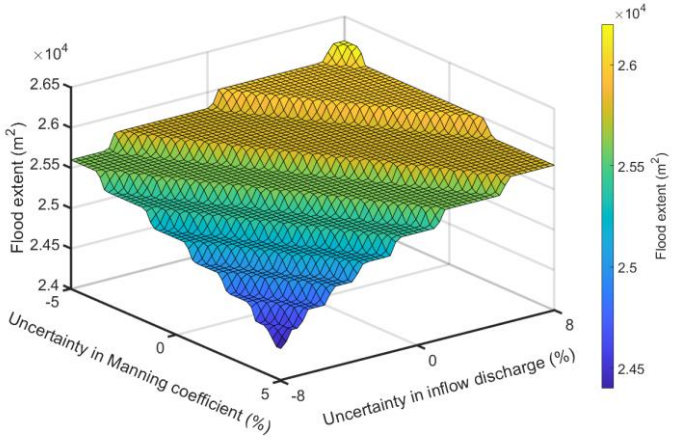
3) Carlisle 2005 flood

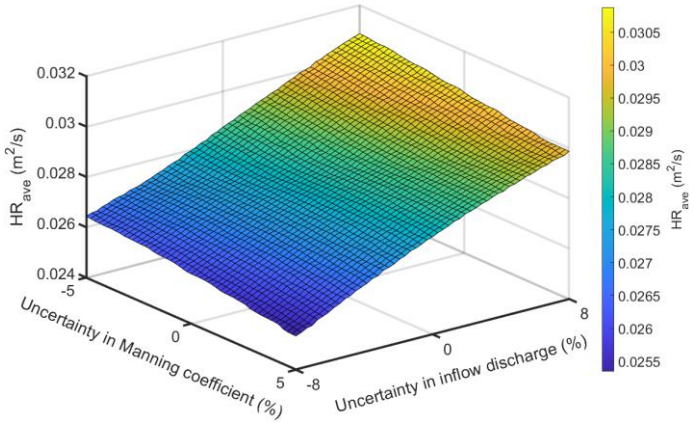


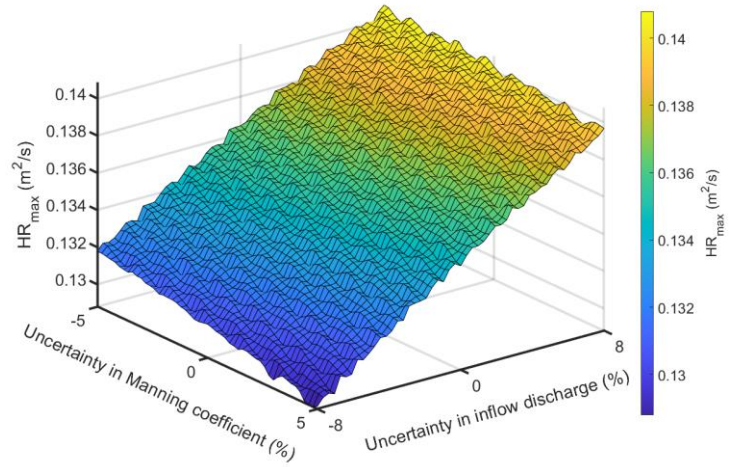
The best



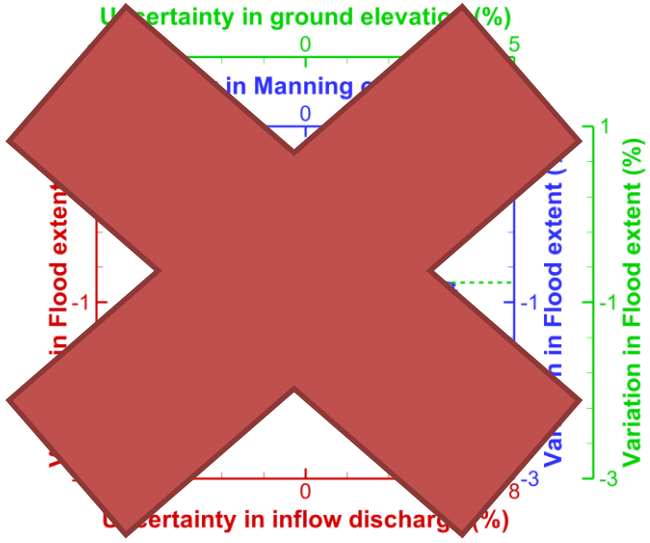
Uncertainty propagation

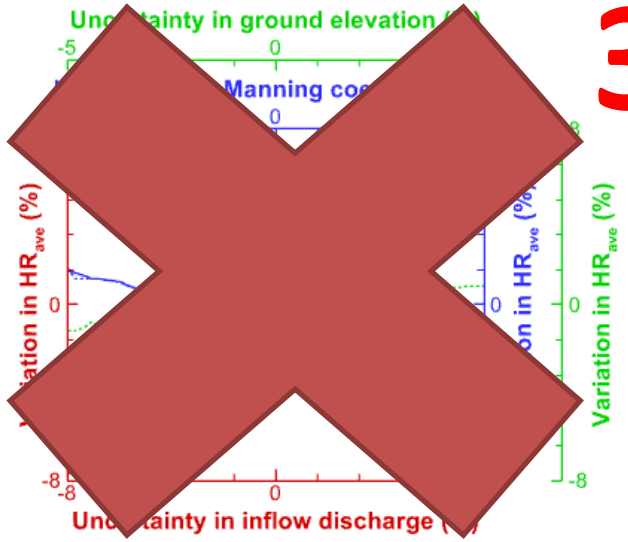




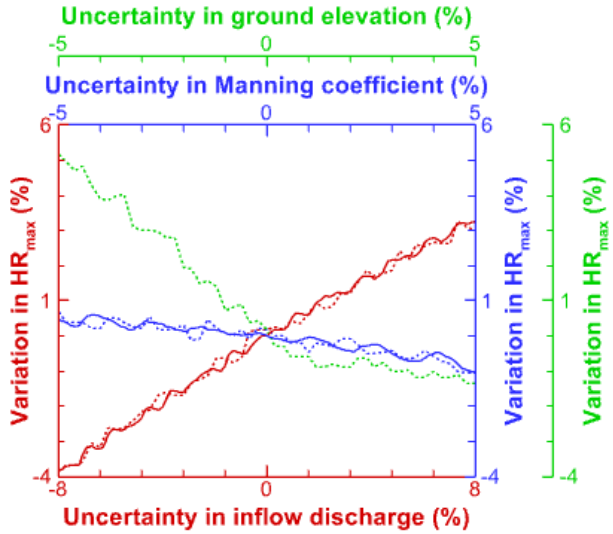


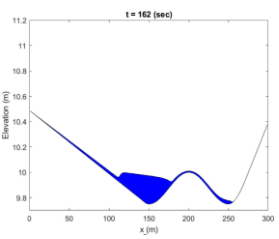
Response in the 2D (straight lines) and 3D cases (broken lines)



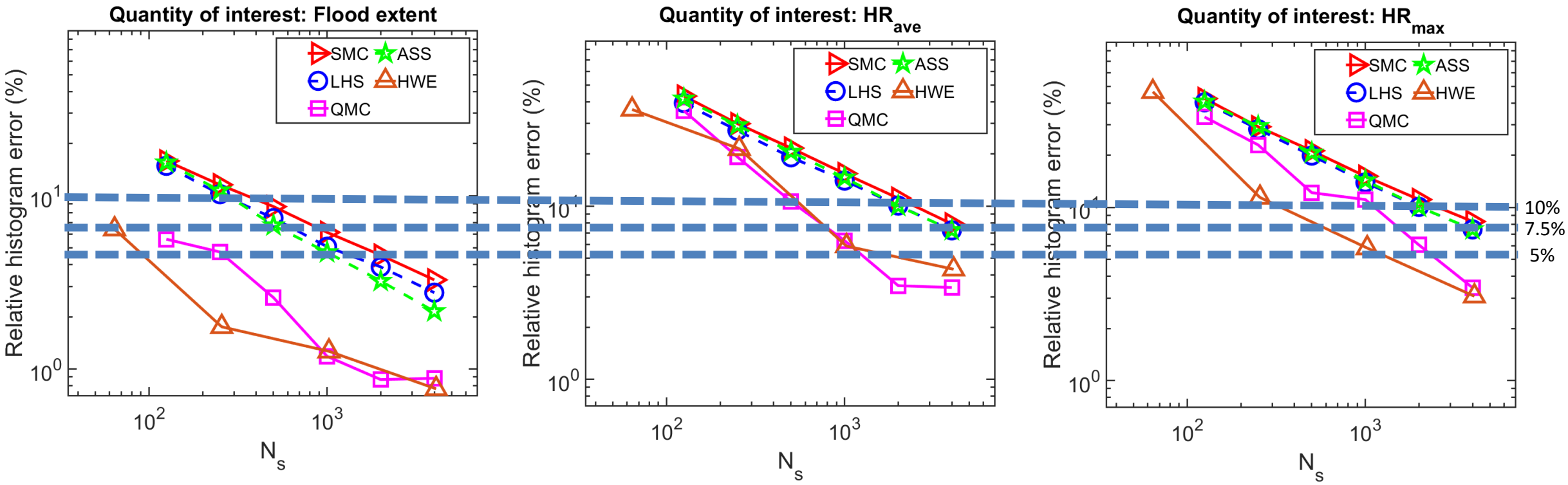


3D



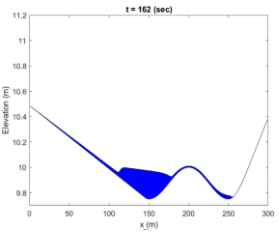


2D: errors and speedups



Average Speedups N_s^{SMC}/N_s^{method} to achieve an error range between 5% and 10%

✓LHS	1.27
✓ASS	1.30
✓QMC	5.57
✓HWE	8.10

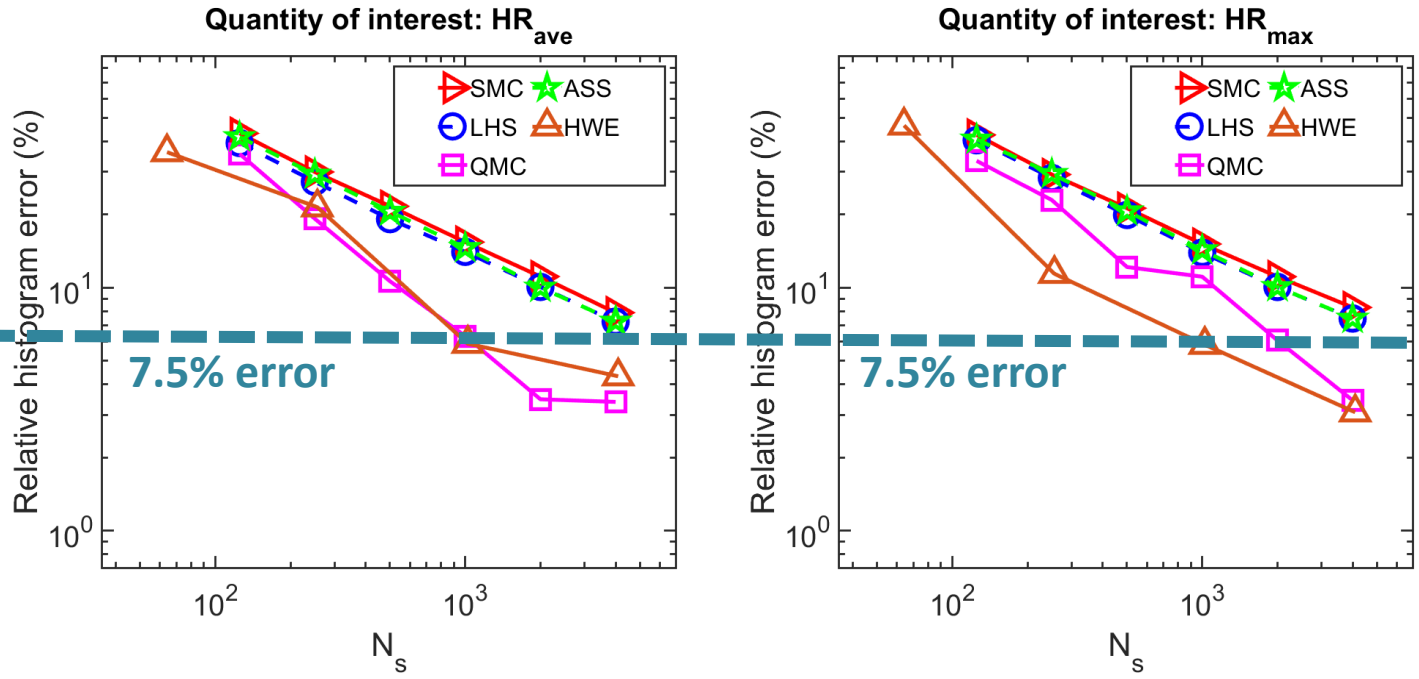


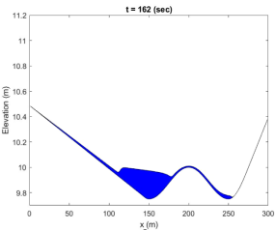
2D: selected sample sizes

SMC (N_s = 4675)

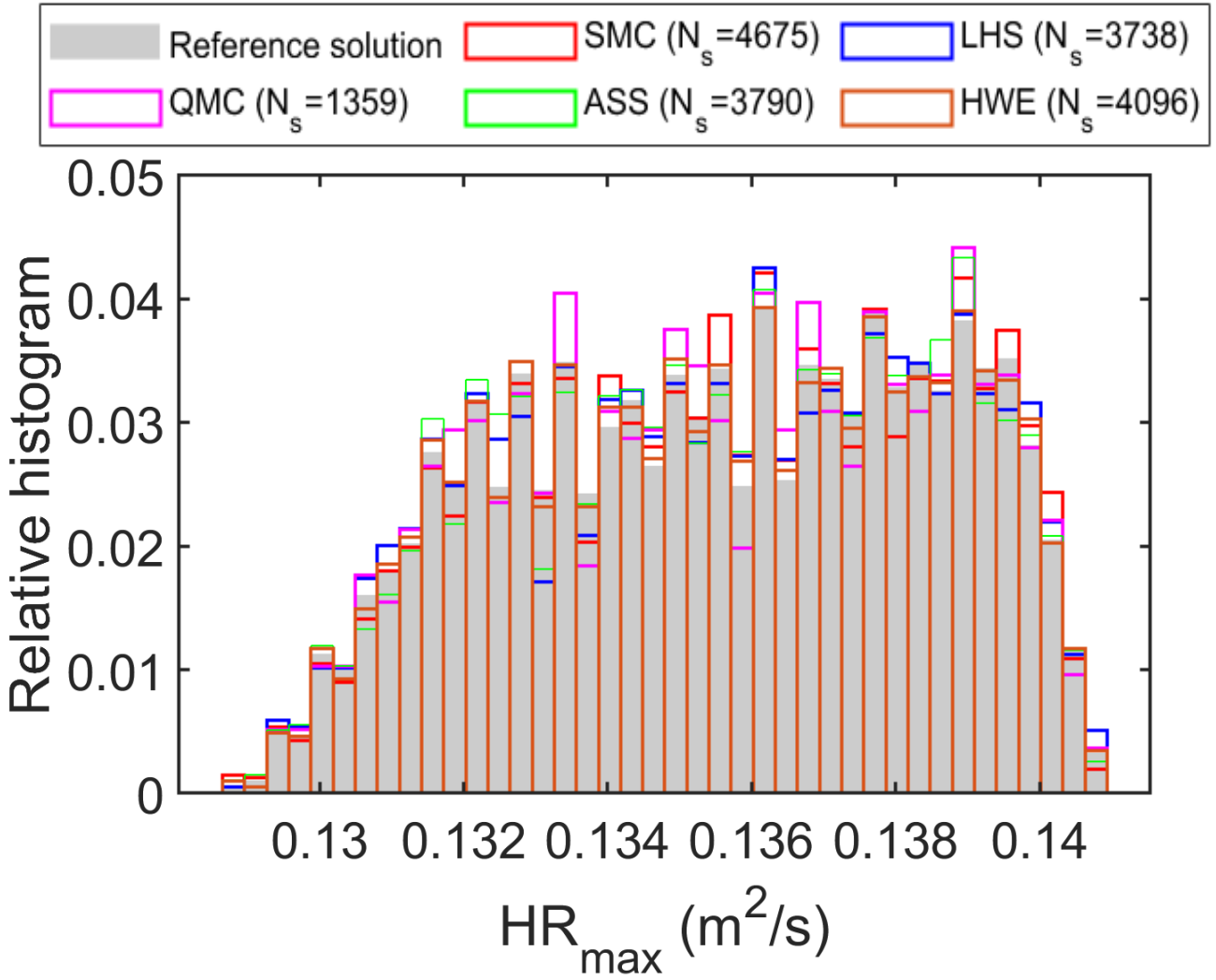
LHS (N_s = 3738) ASS (N_s = 3790)

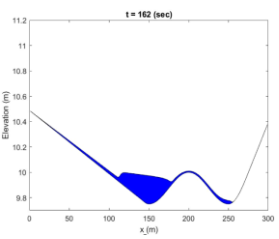
QMC (N_s = 1359) HWE (N_s = 4096)



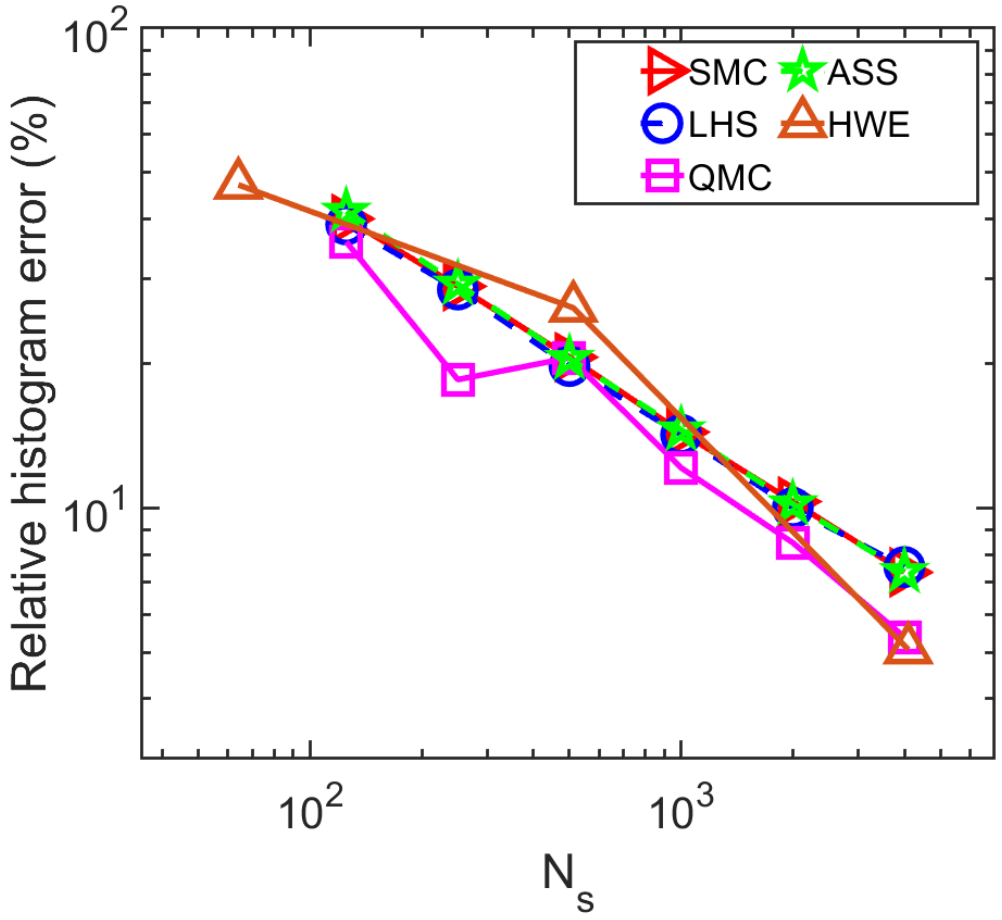


2D: sampled histograms



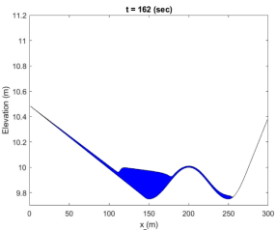


3D: errors and speedups

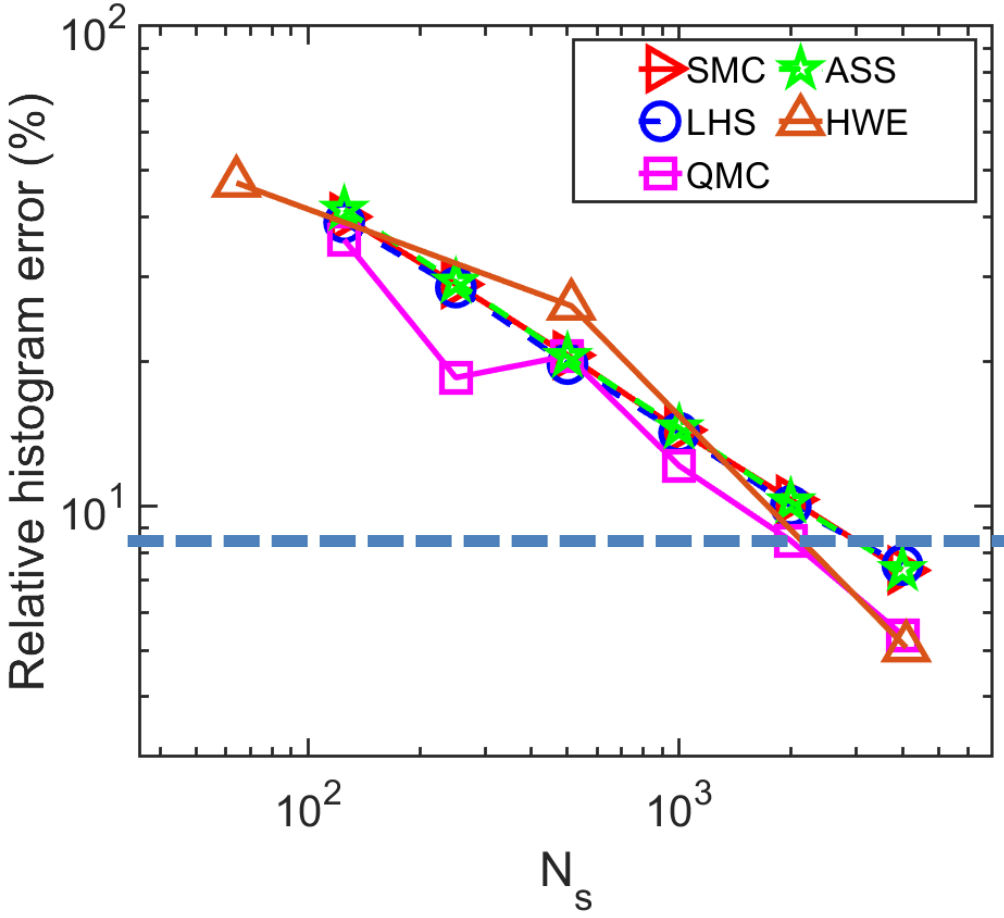


Average Speedups

✓LHS	1.00
✓ASS	1.02
✓QMC	1.59
✓HWE	1.19



3D: selected sample sizes



SMC (N_s = 3823)

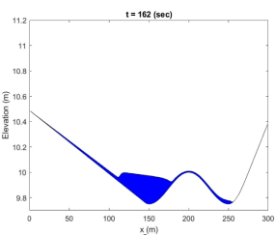
LHS (N_s = 3821)

ASS (N_s = 3766)

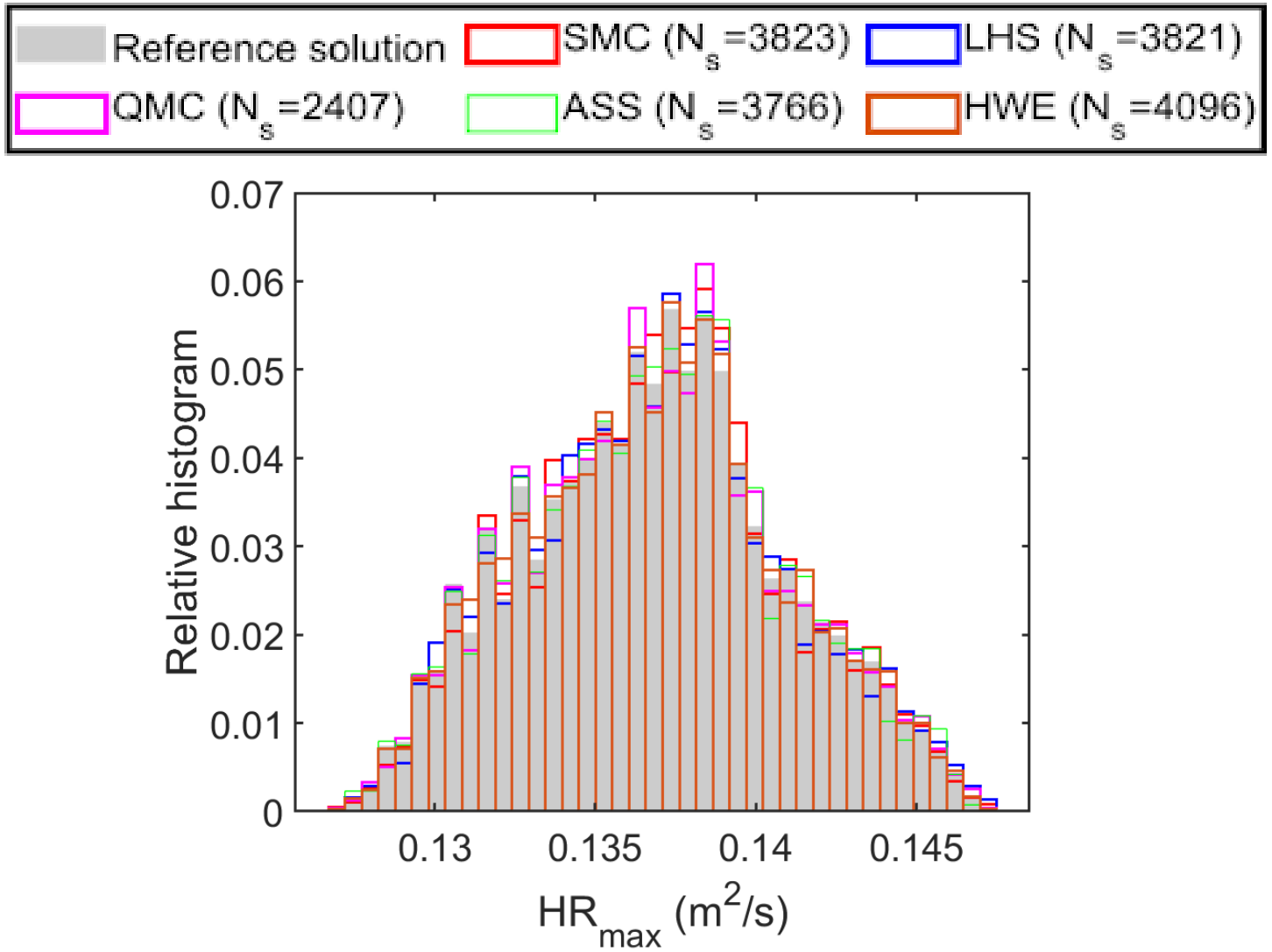
HWE (N_s = 4096)

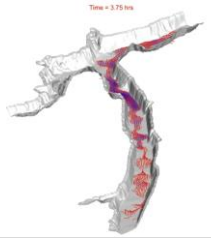
QMC (N_s = 2407)

7.5% error

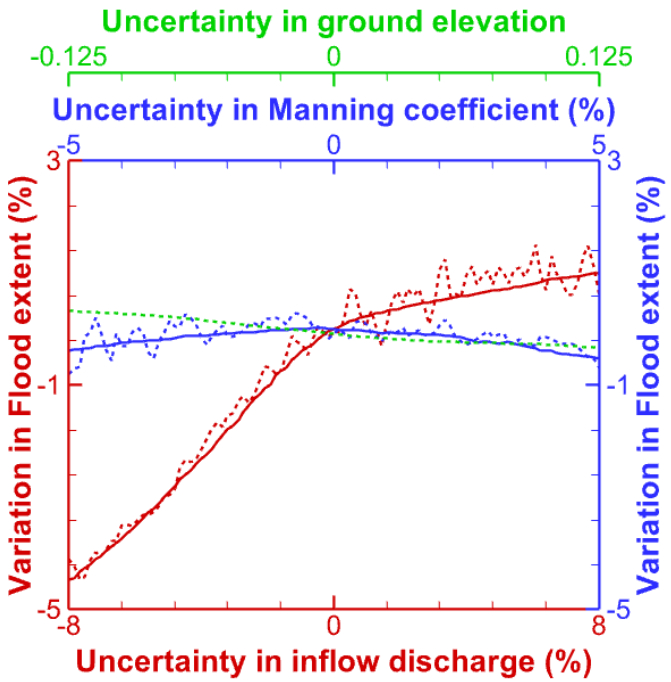


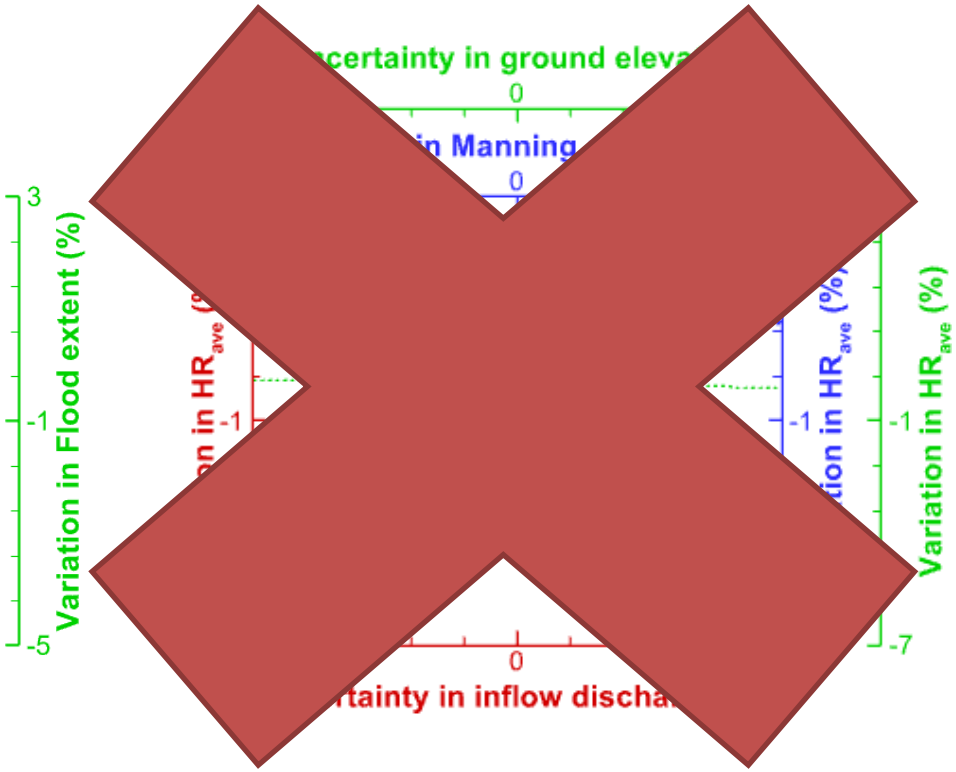
3D: sampled histogram

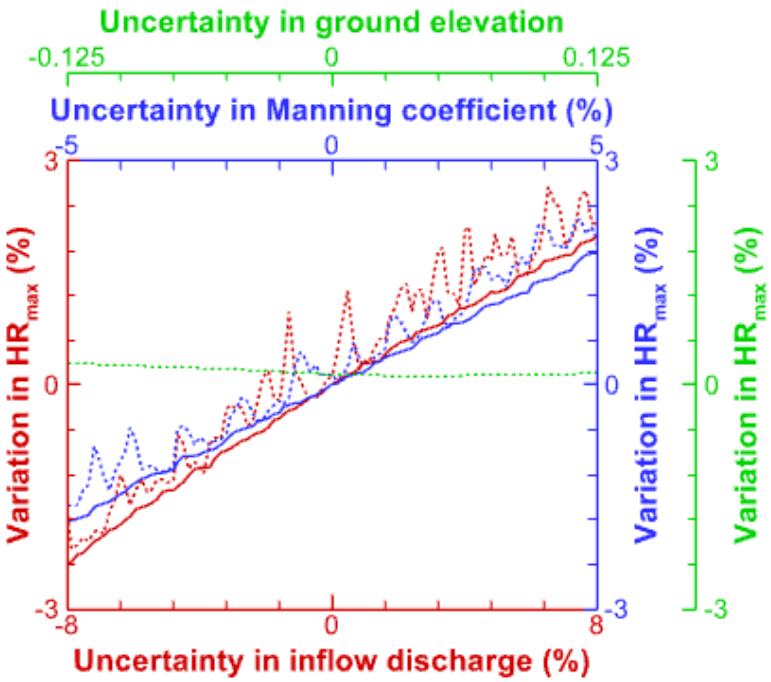




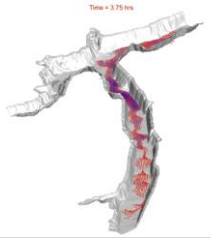
Uncertainty propagation



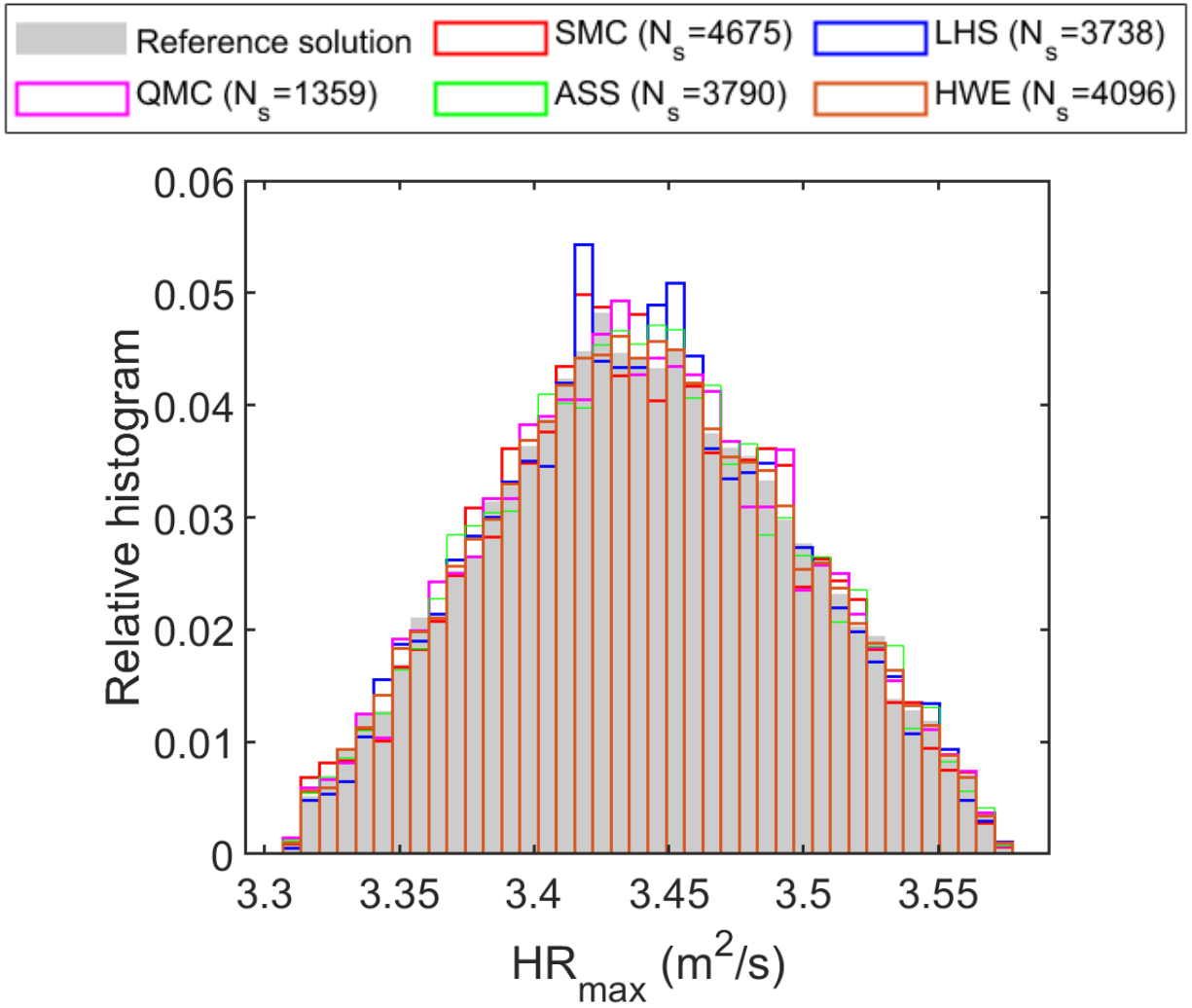


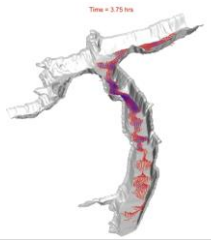


3D



2D: sampled histograms

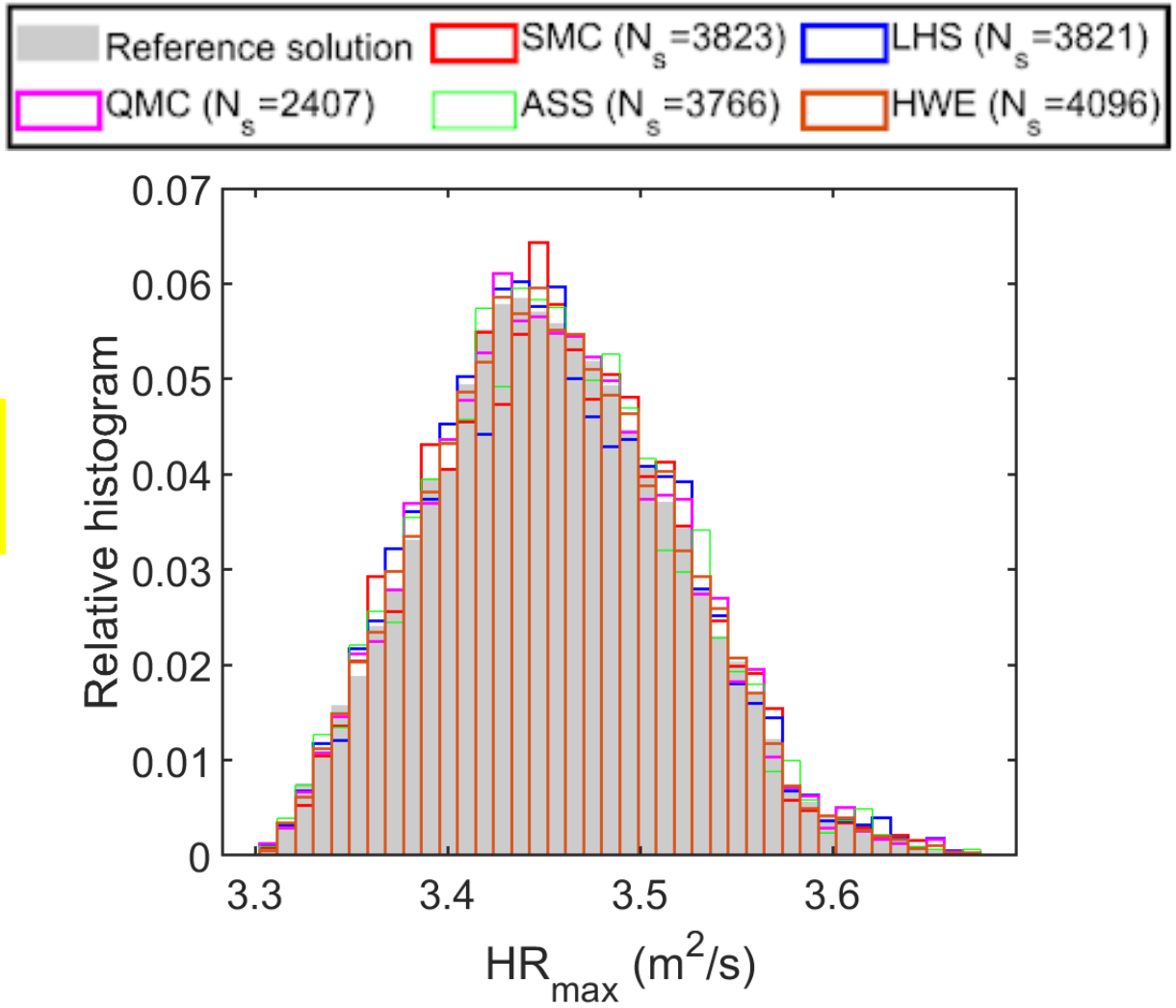


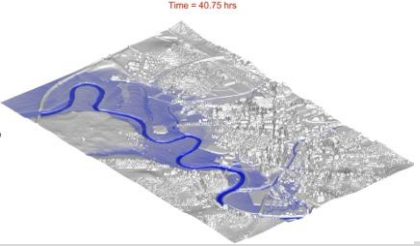


3D: sampled histograms

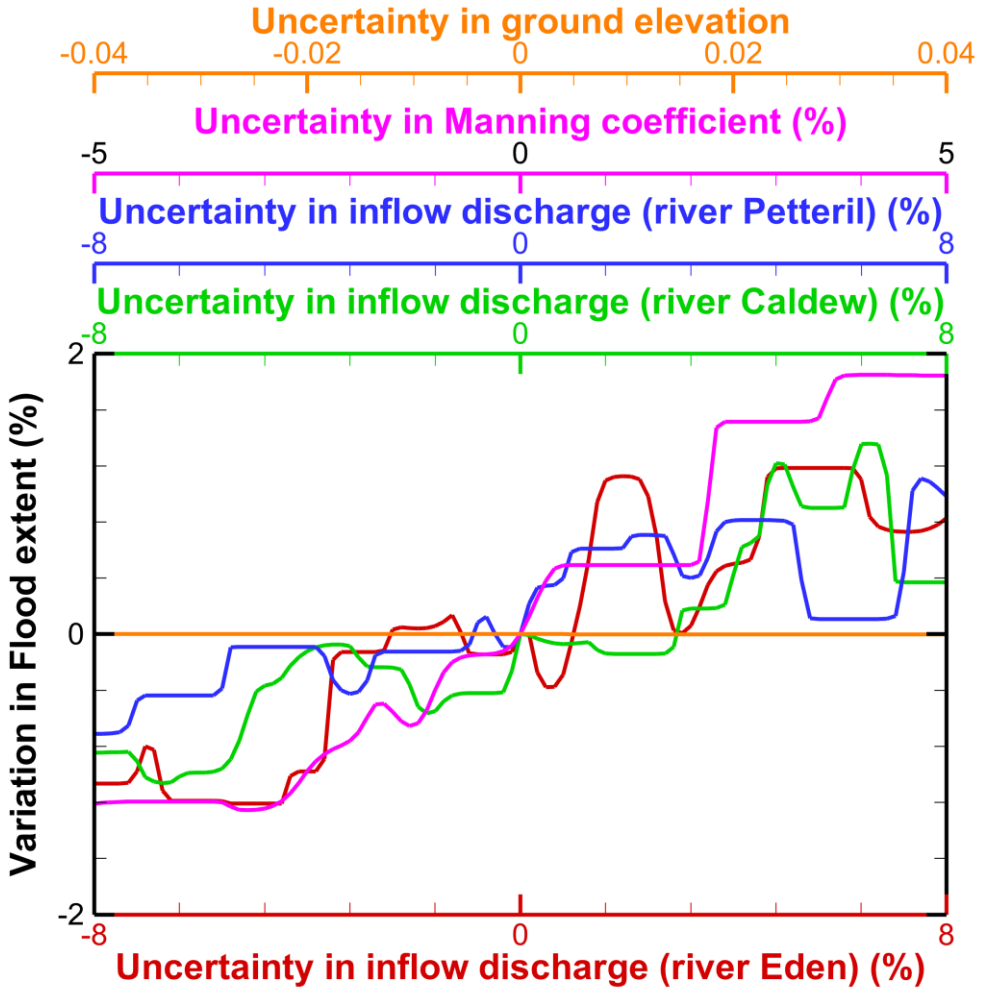
ASS

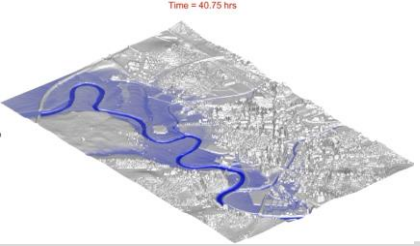
QMC





Uncertainty propagation





5D: histograms